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AN INVERSE OF THE INCOMPLETE BETA FUNCTION (F-(VARIANCE RATIO) DISTRIBUTION FUNCTION)

BY ARMIDO DIDONATO

FORCE WARFARE SYSTEMS DEPARTMENT

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13. ABSTRACT (Maximum 200 words)

This report describes an algorithm, IBETA, to evaluate an inverse of the Incomplete Beta Function or, equivalently, an inverse of the F–Distribution Function.

A Fortran 95 double-precision subroutine, INVBETA, is available that is based on IBETA. It produces the inverse up to 10 significant digits whenever the word length of the computer used is not limiting.

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FOREWORD

This report contains the description of an algorithm that is the basis for the Fortran software of an important statistical function used in targeting studies. The software satisfies the high standards required for its inclusion in the NSWC Library of Mathematics Subroutines.

Dr. John Crigler (B10) supplied the external distribution list.

The editorial assistance of David Bozicevich (B60) is appreciated.

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I. INTRODUCTION

The Incomplete Beta Function, $\mathrm{I_x(a,b)}$ [1, p. 263, p. 944], is defined by

$$\mathrm{I_x(a,b) = G(a,b)\int_0^x t^{a-1}\,(1-t)^{b-1}\,dt, \qquad a>0,\;\; b>0,\;\; 0\le x\le 1} \tag{1}$$

$$\mathrm{B(a,b) = 1/G(a,b) = \int_0^1 t^{a-1}\,(1-t)^{b-1}\,dt = \Gamma(a)\,\Gamma(b)/\Gamma(a+b),} \tag{2}$$

where the gamma function $\Gamma(\mathrm{u})$ is given by

$$\Gamma(\mathrm{u}) \equiv \int_0^\infty \mathrm{e^{-t}\,t^{u-1}\,dt, \quad u>0, \qquad [1, p.\,255].} \tag{3}$$

The complement of $\mathrm{I_x(a,b)}$ is given by

$$\mathrm{I_y(b,a) = 1 - I_x(a,b), \qquad y = 1-x.} \tag{4}$$

$\mathrm{I_x(a,b)}$ is numerically evaluated by the Fortran 95 subroutine called DBRAT contained in the NSWC Mathematics Library [7, p. 88]. It requires $\mathrm{a, b, x, y}$ as input, where both x and y are specified, so that a value for either x or y that is smaller than ϵ can be used. Here ϵ is the smallest positive double-precision number for the computer in use such that $1+\epsilon > 0$ (see [2], [3]). In our use of an IBM PC, $\epsilon = 2^{-52} \simeq 2.220446\mathrm{E}(-16)$.

$\mathrm{I_x(a,b)}$ appears in many branches of science, including atomic physics, fluid dynamics, transmission theory, lattice theory, and operations research [4]. It is perhaps best known in statistics by its direct connection to the F−Distribution, $\mathrm{P(F_0|\nu_1,\,\nu_2)}$ [1, p. 946]. The F−Distribution and its complement are given by

$$\mathrm{P(F_0|\nu_1,\,\nu_2) = \nu_1^{\nu_1/2}\,\nu_2^{\nu_2/2}\,G(\nu_1/2,\nu_2/2)\int_0^{F_0} F^{(\nu_1-2)/2}(\nu_2+\nu_1\,F)^{-(\nu_1+\nu_2)/2}\,dF,} \tag{5}$$

$$\mathrm{Q(F_0|\nu_1,\,\nu_2) = 1 - P(F_0|\nu_1,\,\nu_2).} \tag{6}$$

Let

$$\mathrm{\nu_1 = 2a, \quad \nu_2 = 2b, \quad F = b(1-t)/(a\,t), \quad F_0 = (b\,y)/(a\,x).} \tag{7}$$

Then, it easily follows that $\mathrm{x = b/(b+aF_0)}$, $\mathrm{y = (aF_0)/(b+aF_0)}$ and

$$\mathrm{P(F_0|\nu_1,\,\nu_2) = I_y(a,b), \qquad Q(F_0|\nu_1,\,\nu_2) = I_x(b,a).} \tag{8}$$

With $\mathrm{a = 1/2}$, $\nu = \nu_2$, the F−Distribution reduces to the Student′s t − Distribution, $(\mathrm{A(t_0|\nu)}$ [1, p. 948]. Thus

$$\mathrm{P(|t|\le t_0) = A(t_0|\nu) = \frac{2}{\sqrt{\nu}}\,G\left(\frac{1}{2},\,\frac{\nu}{2}\right)\int_0^{t_0}(1+t^2/\nu)^{-(\nu+1)/2}\,dt = I_y(1/2,b),} \tag{9}$$

where

$$t_0 = \sqrt{F_0}\,, \qquad y = t_0^2/(\nu + t_0^2). \tag{10}$$

In addition, $I_x(a,b)$ is also related to the Binomial Distribution, E [1, p. 960], namely

$$E(n,r,x) = \sum_{i=r}^{n} \binom{n}{i} x^i\,(1-x)^{n-i} = I_x(r, n-r+1). \tag{11}$$

The objective in this report is to give a numerical algorithm to obtain the inverse of I_x, that is, to find x (and y) given a, b, I_x, I_y. In addition, a Fortran 95 computer program will be described that determines the smaller of x and y within a prescribed number of significant digits. For reasons that required the input of both x and y for DBRAT [2], we will, similarly, now require both I_x and I_y as input.

II. ALGORITHM FOR x AND y

In order to obtain values of x and y with a prespecified relative accuracy, the smaller of the two is always the one computed, and the other is determined as its one's complement. Also, in referring to $I_x(a,b)$ or $I_y(b,a)$, it is usually the smaller that is used. Note that $I_x(a,b) < I_y(b, a)$ does not necessarily imply that $x < y$. See the example directly following (18). Therefore let I_{zn} (I_{zo}) denote the new (previous) iterate for I_z, where z = x or y or xy, with

$$xy \equiv \min(x,y), \qquad I_{xy} \equiv \min(I_x,\, I_y), \qquad I_{xyn} \equiv \min(I_{xn},\, I_{yn}) \qquad I_{xyo} \equiv \min(I_{xo},\, I_{yo}). \tag{12}$$

Also let

$$\begin{array}{lcllcl} \text{Previous iterate for x} & \equiv & \text{xo.} & \text{New iterate for x} & \equiv & \text{xn.} \\ \text{Previous iterate for y} & \equiv & \text{yo.} & \text{New iterate for y} & \equiv & \text{yn.} \\ \text{Previous iterate for xy} & \equiv & \text{xyo.} & \text{New iterate for xy} & \equiv & \text{xyn.} \end{array} \tag{13}$$

The numerical evaluation of xy from I_{xy} for given a and b is essentially an iterative rootfinding process. For this purpose, the Newton-Raphson (N-R) procedure [5, p. 129] worked very well. By this we mean that it was successfully used to find xy numerically over a large range of input. However, before one can use N-R with assurance, a so-called "domain of attraction" (DOA) must be determined, which contains an estimate for xy from which a few N-R iterations will converge to an acceptable approximation for xy. Extensive testing determined that if I_{xyn} satisfies

$$|I_{xyn} - I_{xy}| < \text{eps1} * I_{xy}, \qquad \text{eps1} = 1(-2)^1, \tag{14}$$

then xyn is in DOA. With three independent variables, a, b, I_{xy}, this is the difficult phase of the analysis.

[1] $A(-B) \equiv A * 10^{-B}$, A and B positive numbers.

For our purposes it is useful to always have an upper, xyH, and lower, xyL, bound for xy such that

$$\mathrm{xyL} < \mathrm{xy} < \mathrm{xyH}. \tag{15}$$

Initially, we start with xyL = 0 and xyH = 1. The first approximation for xy is given by either

$$\mathrm{xo} = [\mathrm{a}\, \mathrm{I_x}\, \mathrm{B(a,b)}]^{1/\mathrm{a}}, \qquad \mathrm{a} \le \mathrm{b}, \tag{16}$$

or

$$\mathrm{yo} = [\mathrm{b}\, \mathrm{I_y}\, \mathrm{B(a,b)}]^{1/\mathrm{b}}, \qquad \mathrm{b} < \mathrm{a}. \tag{17}$$

Estimate (16) is particularly good for small a, and (17) for small b. The heuristic argument for (16) is based on the relationship [1, p. 944]

$$\mathrm{I_x(a,b)} = \frac{\mathrm{G(a,b)}}{\mathrm{a}} \mathrm{x^a (1-x)^b} + \mathrm{I_x(a+1,b)}. \tag{18}$$

Since $\mathrm{I_x(a,b)} \to 1$ as $\mathrm{a} \to 0$, then for small a and reasonable values of $\mathrm{I_x}$, x will be small so that the last term in (18) is negligible relative to the first, thus yielding (16). For example, given $\mathrm{I_x} = .9$ and $\mathrm{a} = .01, \mathrm{b} = 2$ (from(16)), $\mathrm{xo} \simeq 9.80047(-6)$ with $\mathrm{x} = 9.801426(-6)$. An argument similar to the one above leads to (17).

Improved estimates for xy are attempted by the following: If

$$\mathrm{I_{xyo}} < \mathrm{I_{xy}} \qquad \text{(see (13))}, \tag{19}$$

then $\mathrm{xyn} = 2 * \mathrm{xyo} < 1$. This doubling procedure is continued for a maximum of 12 cycles as long as (19) is satisfied with a resulting improvement in xyL. If (19) is not satisfied initially, then xyn = xyo/2. This halving procedure is continued for a maximum of 12 cycles as long as (19) is not satisfied with a resulting improvement in xyH.

At this stage xyn is assigned,

$$\mathrm{xyn} = (\mathrm{xyH} + \mathrm{xyL})/2, \tag{20}$$

so that an improved value of xyH or xyL is found depending on whether $\mathrm{I_{xyn}}$ is greater or less than $\mathrm{I_{xy}}$. This procedure is cycled using (20) a maximum of 35 times unless, at some stage, (14) is satisfied or

$$|\mathrm{xyH} - \mathrm{xyL}| < \mathrm{eps2} * \mathrm{xyn}, \quad \mathrm{eps2} = 1(-13). \tag{21}$$

If (21) holds, then xyn is accepted as a satisfactory result for xy.

If only (14) holds, then xyn is in DOA and the N-R procedure is called. Setting xyn to xyo (see (13)), the new estimate for xy is computed from (N-R) using the following algorithm (see next page).

Let

$$D_{xyo} = \begin{cases} xo^{a}\,(1-xo)^{b}/(\,B(a,b)\,xo\;yo\,) & \text{for } xo \le yo \\ yo^{b}\,(1-yo)^{a}/(\,B(a,b)\,xo\;yo\,) & \text{for } yo < xo. \end{cases} \tag{22}$$

Now:

$$\begin{array}{l}
\text{If } I_x \le I_y \text{ then} \\
\quad w = (I_{xyo} - I_x)/D_{xyo}, \\
\quad \text{If } xo \le yo \text{ then} \\
\quad\quad xyn = xn = xo - w \\
\quad \text{Else} \\
\quad\quad xyn = yn = yo + w \\
\quad \text{Endif} \\
\text{Else } (I_y < I_x) \\
\quad w = (I_{xyo} - I_y)/D_{xyo}, \\
\quad \text{If } yo \le xo \text{ then} \\
\quad\quad xyn = yn = yo - w \\
\quad \text{Else} \\
\quad\quad xyn = xn = xo + w \\
\quad \text{Endif} \\
\text{Endif}
\end{array} \tag{23}$$

The N-R procedure is cycled until

$$|w| \le eps * xyn, \tag{24}$$

where eps is assigned by the user. An eps = 5(−10) requires, for convergence, no more than 4 N-R iterations for a and/or b as large as 1(8) and I_{xy} as small as 1(-10).

III. COMPUTER PROGRAM FOR x and y–INVBETA (a, b, Ix, Iy, x, y, eps, Ind)

INVBETA (a, b, Ix, Iy, x, y, eps, Ind) denotes the Fortran 95 [6] subroutine with its call line. All variables in the call line are specified double-precision, except the integer Ind. The inputs are a, b, I_x, I_y and eps. These have been defined in the previous sections. The outputs are x, y and Ind. The input eps assigns the accuracy desired in x and y. If a relative error less than 1(-N) is desired in x and y, then eps = 5(-N-1). The value of N is limited to a positive integer no larger than 10. The integer output Ind specifies whether the output x and y are acceptable. The tablulation below indicates the values Ind can take and their meanings.

Ind = 0, Results for x and y are acceptable.

Ind = 1, $a \le 0$ and/or $b \le 0$ are unacceptable, $a > 0$, $b > 0$ required.

Ind = 2, $I_x + I_y \ne 1$ is unacceptable, $I_x + I_y = 1$ required.

Ind = 3, $Ix < 0$ or $Ix > 1$ are not acceptable.

Ind = 4, Iy < 0 or Iy > 1 are not acceptable.

Ind = 100, x is estimated to be less that 1(-200).

Ind = −100, y is estimated to be less that 1(-200).

Ind = 10, More than 20 iterations of N-R required on x, returns to halving stage.

Ind = 11, More than 20 iterations of N-R required on y, returns to halving stage.

The Fortran source file IBETA.FOR contains INVBETA and 43 supporting routines from [7].

IV. NUMERICAL RESULTS

In Table 1 below, the first four columns contain inputs for INVBETA. The next two columns contain outputs x, y, and the last column contains the total number of calls to DBRAT. A row of data refers to one case, or call to INVBETA. There are 23 cases listed. The accuracy parameter was set at eps $= 5(-11)$.

TABLE 1. NUMERICAL RESULTS

1(-2)	10	.99	1(-2)	.2748462647(-1)	.9725153735	7
10	.10	.9999	1(-4)	1.0	.6356342847(-41)	3
2	5	.1	.9	.9259525891(-1)	.9074047411	11
2	5	.5	.5	.2644499833	.7355500167	8
2	5	.9	.1	.5103163066	.4896836934	9
2	5	.999	1(-3)	.8186138669	.1813861331	9
10	25	.1	.9	.1914227060	.8085772940	8
10	25	.5	.5	.2815933420	.7184066580	10
10	25	.9	.1	.3854635784	.6145364216	12
10	25	.999	1(-3)	.5413352819	.4586647181	14
50	125	.1	.9	.2425526829	.7574473171	13
50	125	.5	.5	.2848963507	.7151036493	12
50	125	.9	.1	.3299331965	.6700668035	15
50	125	.999	1(-3)	.3969077527	.6030922473	11
500	925	.1	.9	.3347225032	.6652774968	14
500	925	.5	.5	.3508074095	.6491925905	12
500	925	.9	.1	.3671216035	.6328783965	15
500	925	.999	1(-3)	.3904834123	.6095165877	16
50000	2500000	.1	.9	.1949665343(-1)	.9805033466	19
50000	2500000	.5	.5	.1960771754(-1)	.9803922825	14
50000	2500000	.9	.1	.1971919420(-1)	.9802808058	17
50000	2500000	.999	1(-3)	.1987722641(-1)	.9801227736	20
50000	2500000	.9999999999	1(-10)	.2016513068(-1)	0.9798348693	21

V. REFERENCES

1. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, Edited by M. Abramowitz and I. Stegun, Applied Mathematics Series 55, June 1955.

2. DiDonato, A.R. and Morris, A.H., *Significant Digit Computation of the Incomplete Beta Function Ratios*, NSWC TR 88-365, November 1988, Naval Surface Warfare Center, Dahlgren, VA 22448.

3. DiDonato, A.R., and Morris, A.H., Significant Digit Computation of the Incomplete Beta Function Ratios, ACM TOMS, Vol. 18, #3, September 1992, pp. 360-373.

4. Dutka, J., The Incomplete Beta Function - A Historical Profile, *Archive for History of Exact Sciences,* 24,1981, pp. 11-29.

5. Johnson, L.W. and Riess, R.D., Numerical Analysis, Addison-Wesley Publishing Co., 1977.

6. Lahey Computer Systems, Inc., 805 Tahoe Boulevard, Incline Village, NV 89450-6091.

7. Morris, A.H., *NSWC Library of Mathematics Subroutines,* NSWCDD/TR-92/425, January 1993, Naval Surface Warfare Center, Dahlgren Division, Dahlgren, VA 22448.

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